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STABILITY OF COMPRESSED VISCOELASTIC ORTHOTROPIC SHELLS

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The stability criterion for structures operating under creep conditions, which is based on comparing the unperturbed and perturbed motion trajectories, is proposed in [1, 2]. On the basis of this criterion, the singularities in the behavior of compressed viscoelastic thin-walled shells are analyzed in this paper.

The equilibrium and strain compatibility equations of thin-walled shallow shells with the interlayer shear taken into account according to the Timoshenko hypothesis are written in the form [3, 4]

$$h[\mathbf{\Gamma}_{3131}(\gamma_{1,1}+u_{3,11})+\mathbf{\Gamma}_{3232}(\gamma_{2,2}+u_{3,22})]+\left[\frac{1}{R_{ij}}+\left(u_3+u_3^0\right)_{,ij}\right]N_{ij}=0,$$

$$\mathbf{D}_{1111}\gamma_{1,11}+\mathbf{D}_{1122}\gamma_{2,12}+\frac{1}{2}\mathbf{D}_{1212}(\gamma_{1,22}+\gamma_{2,12})=h\mathbf{\Gamma}_{3131}(\gamma_1+u_{3,1}),$$

$$\mathbf{D}_{2211}\gamma_{1,12}+\mathbf{D}_{2222}\gamma_{2,22}+\frac{1}{2}\mathbf{D}_{1212}(\gamma_{1,12}+\gamma_{2,11})=h\mathbf{\Gamma}_{3232}(\gamma_2+u_{3,2}), \tag{1}$$

$$\frac{1}{h}[\mathbf{K}_{1111}F_{,2222}+2(\mathbf{K}_{1212}+\mathbf{K}_{1122})F_{,1122}+\mathbf{K}_{2222}F_{,1111}]=-e_{ik}e_{jl}\left\{\frac{1}{R_{kl}}u_{3,ij}+\right.$$
$$\left.+\frac{1}{2}\left[\left(u_3+u_3^0\right)_{,kl}\left(u_3+u_3^0\right)_{,ij}-u_{3,kl}^0u_{3,ij}^0\right]\right\}\qquad(i,j,k,l=1,2),$$

where γ_i are the angles of rotation of the normal to the middle surface; u_3, u_3^0 are the additional and initial shell deflections; F is a function of forces acting in the middle surface, $N_{ij}=e_{ik}e_{jl}F_{,kl}$; h, R_{ij} are the thickness and radii of curvature ($R_{12}=R_{21}=\infty$); K_{ijkl}, Γ_{ijkl} are operators of the form

$$\mathbf{K}_{ijkl}f=\frac{1}{E_{ijkl}}f(t)+\int_0^t K_{ijkl}(t-\tau)f(\tau)\,d\tau,$$

$$\mathbf{\Gamma}_{ijkl}f=c_{ijkl}f(t)-\int_0^t \Gamma_{ijkl}(t-\tau)f(\tau)\,d\tau;$$

E_{ijkl}, c_{ijkl} are elastic constants; $K_{ijkl}(t-\tau)$, $\Gamma_{ijkl}(t-\tau)$ are the creep and relaxation kernels which are invariant relative to the origin:

$$0\leqslant\int_0^\infty K_{ijkl}(\tau)\,d\tau=K_{ijkl}<\infty,\quad 0\leqslant\int_0^\infty \Gamma_{ijkl}(\tau)\,d\tau=\Gamma_{ijkl}<1,$$

$$\mathbf{D}_{ijkl}=\frac{h^3}{12}\mathbf{\Gamma}_{ijkl},\quad e_{ik}=\begin{cases}1, & i>k\\ 0, & i=k\\ -1, & i<k.\end{cases}$$

Here and henceforth the summation is over the repeated subscripts. The subscripts following the comma denote differentiation with respect to the appropriate coordinate. The x_1, x_2 axes coincide with the lines of principal curvature and the axes of viscoelastic symmetry, while the x_3 axis is perpendicular to them and directed toward the center of curvature.

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Let us assume that small initial perturbations in the shell deflection δu_3^0 occur in addition to the deflection u_3^0. The perturbations in the additional deflection δu_3, the angles of normal rotation $\delta\gamma_i$, and the force function δF are found from the equations

$$
\begin{gathered}
h\left[\mathbf{\Gamma}_{3131}(\delta\gamma_{1,1}+\delta u_{3,11})+\mathbf{\Gamma}_{3232}(\delta\gamma_{2,2}+\delta u_{3,22})\right]+\\
+\left[\frac{1}{R_{ij}}+(u_3+u_3^0)_{\cdot ij}\right]\delta N_{ij}+N_{ij}(\delta u_3+\delta u_3^0)_{\cdot ij}=0,\\
\mathbf{D}_{1111}\delta\gamma_{1,11}+\mathbf{D}_{1122}\delta\gamma_{2,12}+\frac{1}{2}\mathbf{D}_{1212}(\delta\gamma_{1,22}+\delta\gamma_{2,12})=h\mathbf{\Gamma}_{3131}(\delta\gamma_1+\delta u_{3,1}),\\
\mathbf{D}_{2211}\delta\gamma_{1,12}+\mathbf{D}_{2222}\delta\gamma_{2,22}+\frac{1}{2}\mathbf{D}_{1212}(\delta\gamma_{1,12}+\delta\gamma_{2,11})=h\mathbf{\Gamma}_{3232}(\delta\gamma_2+\delta u_{3,2}),\\
\frac{1}{h}\left[\mathbf{K}_{1111}\delta F_{\cdot 2222}+2(\mathbf{K}_{1212}+\mathbf{K}_{1122})\delta F_{\cdot 1122}+\mathbf{K}_{2222}\delta F_{\cdot 1111}\right]=\\
=-e_{ik}e_{jl}\left\{\frac{1}{R_{kl}}\delta u_{3,ij}+\frac{1}{2}\left[(u_3+u_3^0)_{\cdot kl}(\delta u_3+\delta u_3^0)_{\cdot ij}+\right.\right.\\
\left.\left.+(\delta u_3+\delta u_3^0)_{\cdot kl}(u_3+u_3^0)_{\cdot ij}-u_{3,kl}^0\delta u_{3,ij}^0-\delta u_{3,kl}^0 u_{3,ij}^0\right]\right\}.
\end{gathered}
\tag{2}
$$

Let us consider a circular cylindrical shell, hinge-clamped along the ends and compressed uniformly in the axial direction by a load q. Let us assume that

$$u_3^0=\sum_m h\xi_m^0\sin\frac{m\pi}{l}x_1 \tag{3}$$

(l is the shell length and the x_1 axis is directed along the generatrix). We have the equations

$$
\begin{gathered}
\mathbf{D}_{1111}u_{3,1111}+\mathbf{D}_{1111}\mathbf{K}_{3131}\frac{1}{h}\left[-q(u_3+u_3^0)_{\cdot 1111}-\frac{h}{R^2}\mathbf{K}_{2222}^{-1}u_{3,11}\right]+q(u_3+u_3^0)_{\cdot 11}+\frac{h}{R^2}\mathbf{K}_{2222}^{-1}u_3=0,\\
F_{\cdot 11}=-\frac{h}{R}\mathbf{K}_{2222}^{-1}u_3
\end{gathered}
\tag{4}
$$

to determine u_3 and F from the system (1). It is evident that

$$u_3=\sum_m h\xi_m\sin\frac{m\pi}{l}x_1.$$

Let us assume that

$$\delta u_3^0(x_1,x_2)=\sum_k c_k^0\sin\frac{k\pi}{l}x_1\cos\frac{n}{R}x_2. \tag{5}$$

Let us seek δu_3, $\delta\gamma_1$, $\delta\gamma_2$ in the form

$$
\begin{gathered}
\delta u_3=\sum_k c_k\sin\frac{k\pi}{l}x_1\cos\frac{n}{R}x_2,\\
\delta\gamma_1=\sum_k\Gamma_k^{(1)}\frac{k\pi}{l}\cos\frac{k\pi}{l}x_1\cos\frac{n}{R}x_2,\\
\delta\gamma_2=\sum_k\Gamma_k^{(2)}\frac{n}{R}\sin\frac{k\pi}{l}x_1\sin\frac{n}{R}x_2.
\end{gathered}
$$

Solving (2), we obtain

$$a_{sn}(1-\alpha_{sn})c_s+\sum_{k,m}\left[\cos(s+k+m)\pi-1\right]\frac{4sm^2n^2h}{\pi R}\left\{\left[\frac{k}{m}\left(\frac{1}{E}\mathbf{K}_{2222}^{-1}\xi_m\right)c_k+\right.\right.$$

$$+\frac{m}{k}\left(\xi_m+\xi_m^0\right)\left(A_{kn}^{-1}c_k\right)\Big]\frac{1}{(s+k+m)(s-k+m)(s+k-m)(s-k-m)}+$$

$$+\frac{1}{4}\left[\frac{(m+k)^2}{k^4(s+k+m)(s-k-m)}B_2^{-1}\left(\left(\xi_m+\xi_m^0\right)c_k\right)-\right.$$

$$\left.-\frac{(m-k)^2}{k^4(s-k+m)(s+k-m)}B_1^{-1}\left(\left(\xi_m+\xi_m^0\right)c_k\right)\right]\Big\}+$$

$$+\sum_{k,m,i}\left(\xi_m+\xi_m^0\right)\left(\frac{n^2mih}{2Rk^2}\right)^2\{B_2^{-1}\left[\left(\xi_i+\xi_i^0\right)c_k\right]T_{skmi}-$$

$$-B_1^{-1}\left[\left(\xi_i+\xi_i^0\right)c_k\right]G_{skmi}\}=\ldots,$$

$$-\left(\mathbf{D}_{1111}\frac{s^2\pi^2}{l^2}+\frac{n^2}{2R^2}\mathbf{D}_{1212}+h\mathbf{\Gamma}_{3131}\right)\mathbf{\Gamma}_s^{(1)}+$$

$$+\frac{n^2}{R^2}\left(\mathbf{D}_{1122}+\frac{1}{2}\mathbf{D}_{1212}\right)\Gamma_s^{(2)}=h\mathbf{\Gamma}_{3131}c_s, \qquad (6)$$

$$\frac{s^2\pi^2}{l^2}\left(\mathbf{D}_{1122}+\frac{1}{2}\mathbf{D}_{1212}\right)\mathbf{\Gamma}_s^{(1)}-\left(\mathbf{D}_{2222}\frac{n^2}{R^2}+\frac{s^2\pi^2}{2l^2}\mathbf{D}_{1212}+h\mathbf{\Gamma}_{3232}\right)\Gamma_s^{(2)}=-h\mathbf{\Gamma}_{3232}c_s,$$

where E is the value of a certain reduced elastic constant;

$$A_{kn}=\frac{l^4}{k^4\pi^4}E\left[\mathbf{K}_{1111}\theta_{kn}^4+2\left(\mathbf{K}_{1212}+\mathbf{K}_{1122}\right)\theta_{kn}^2+\mathbf{K}_{2222}\right];$$

$$B_1=\frac{l^4}{k^4\pi^4}E\left[\mathbf{K}_{1111}\theta_{kn}^4+2\left(\mathbf{K}_{1212}+\mathbf{K}_{1122}\right)\left(\frac{m}{k}-1\right)^2\theta_{kn}^2+\mathbf{K}_{2222}\left(\frac{m}{k}-1\right)^4\right];$$

$$B_2=\frac{l^4}{k^4\pi^4}E\left[\mathbf{K}_{1111}\theta_{kn}^4+2\left(\mathbf{K}_{1212}+\mathbf{K}_{1122}\right)\left(\frac{m}{k}+1\right)^2\theta_{kn}^2+\mathbf{K}_{2222}\left(\frac{m}{k}+1\right)^4\right];$$

$$\theta_{kn}=\frac{nl}{k\pi R},\quad s^2-(k+m)^2\neq 0,\quad s^2-(k-m)^2\neq 0;$$

$$G_{skmi}=\int_0^1[\cos(s+k-m-i)\pi x-\cos(s-k+m+i)\pi x+$$

$$+\cos(s-k+m-i)\pi x-\cos(s+k-m+i)\pi x]\,dx;$$

$$T_{skmi}=\int_0^1[\cos(s+k+m-i)\pi x-\cos(s+k+m+i)\pi x+$$

$$+\cos(s-k-m-i)\pi x-\cos(s-k-m+i)\pi x]\,dx;$$

$$a_{sn}(1-\alpha_{sn})c_s=\frac{R^2}{Eh}\left[\mathbf{D}_{1111}\frac{s^4\pi^4}{l^4}\Gamma_s^{(1)}+\frac{s^2\pi^2n^2}{l^2R^2}\left(\mathbf{D}_{1212}+\mathbf{D}_{1122}\right)\left(\Gamma_s^{(1)}-\Gamma_s^{(2)}\right)-\right.$$

$$\left.-\mathbf{D}_{2222}\frac{n^4}{R^4}\Gamma_s^{(2)}-\frac{Eh}{R^2}A_{sn}^{-1}c_s+\frac{s^2\pi^2}{l^2}qc_s\right].$$

The series of dots in the right side of (6) denote components dependent on the quantity c_k^0.

Keeping one member in the sum (3) for a long shell, (6) is written as follows (m is odd, $k\sim m/2$):

$$a_{sn}(1-\alpha_{sn})c_s+\sum_k\frac{n^2h}{\pi R}\left\{\left(\frac{1}{E}\mathbf{K}_{2222}^{-1}\xi_m\right)c_k+4\left(\xi_m+\xi_m^0\right)\left(A_{kn}^{-1}c_k\right)+4B_1^{-1}\left[\left(\xi_m+\xi_m^0\right)c_k\right]\right\}\frac{1}{s+k-m}-$$

$$-\sum_k\left(\frac{2n^2h}{R}\right)^2\left(\xi_m+\xi_m^0\right)\left(B_2^{-1}+B_1^{-1}\right)\left[\left(\xi_m+\xi_m^0\right)c_k\right]\int_0^1\cos(s-k)\pi x\,dx=\ldots, \qquad (7)$$

Replacing the operators Γ_{ijkl}, K_{ijkl} in (6) and (7) by the elastic constants and equating the determinant comprised of coefficients of c_k to zero, we obtain an equation to find the critical value of the deflection ξ_m^0 for a fixed load q and, conversely, the critical value of the load q for a fixed deflection ξ_m^0.

If k and s are close together, then $a_{sn}\approx a$ = const, $\theta_{sn}\approx\theta_{kn}=\theta$, $\alpha_{sn}\approx\alpha$ = const. The determinant of the system of equations (7) then has the form

$$\begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot\ \lambda + {}^1/_7 & {}^1/_5 & {}^1/_3 & 1 & -1 & \cdot \\ \cdot\ {}^1/_5 & \lambda + {}^1/_3 & 1 & -1 & -{}^1/_3 & \cdot \\ \cdot\ {}^1/_3 & 1 & \lambda - 1 & -{}^1/_3 & -{}^1/_5 & \cdot \\ \cdot\ 1 & -1 & -{}^1/_3 & \lambda - {}^1/_5 & -{}^1/_7 & \cdot \\ \cdot\ -1 & -{}^1/_3 & -{}^1/_5 & -{}^1/_7 & \lambda - {}^1/_9 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} = 0, \tag{8}$$

where

$$\lambda = \frac{-a(1-\alpha)+y}{z}; \quad y = \frac{4n^4h^2}{R^2}(B_1^{-1}+B_2^{-1})(\xi_m+\xi_m^0)^2;$$

$$z = \frac{n^2h}{\pi R}\left[\frac{E_{2222}}{E}\xi_m + 8(\xi_m+\xi_m^0)A^{-1}\right].$$

It hence follows that

$$\frac{4n^4h^2}{R^2}(B_1^{-1}+B_2^{-1})(\xi_m+\xi_m^0)^2 - \lambda\frac{n^2h}{\pi R}\left[\frac{E_{2222}}{E}\xi_m + 8(\xi_m+\xi_m^0)A^{-1}\right] - a(1-\alpha) = 0. \tag{9}$$

It is interesting to compare the values of λ_{max} which correspond to different orders of the determinant (8).

We have $\lambda^{I} = 1$; $\lambda^{III} = 1.569$; $\lambda^{V} = 1.571$; $\lambda^{IX} = 3.1415926/2$ from an examination of determinants of 1st, 3rd, 5th, and 9th orders

Values of $\xi_m + \xi_m^o$ calculated from determinants of the system (6) (columns 3, 4, and 5) for an isotropic shell from an incompressible material without taking account of shear of layers with the geometric characteristics $\pi R/l = 1$, $R/h = 147$, $m = 21$ are presented in Table 1 to estimate the error induced by replacing a finite shell by an infinite shell. The values of s and k in Table 1 correspond to the ordinal numbers of the components in the sum (5). The parameter n corresponds to the least value of the dimensionless shell deflection $\xi_m + \xi_m^o$. Values of $\xi_m + \xi_m^o$ found from (9), which becomes in this particular case

$$4\eta[(1+\theta^2)^{-2}+(9+\theta^2)^{-2}](\xi_m+\xi_m^0)^2 - \frac{\lambda}{\pi}[\alpha_m + 8(1+\theta^2)^{-2}](\xi_m +$$

$$+\xi_m^0) - \frac{a(1-\alpha)}{\eta} = 0, \tag{10}$$

where

$$-a = \frac{1}{4}[(1+\theta^2)^2/4 + 4(1+\theta^2)^{-2}],\ \alpha = -\alpha_m/2a,\ \eta = \frac{3}{4}\theta^2, \eta = \frac{n^2h}{R}, \lambda = \frac{\pi}{2}.$$

are contained in column 2.

As is seen from Table 1, the magnitudes of the critical parameters for the two shells are close together. Therefore, in some cases the stability investigation of a shell of finite length can be replaced by the solution of an analogous problem for an infinite shell.

Let us examine three shells possessing different viscoelastic properties.

Example 1. The shell is orthotropic, where the operators Γ_{1111}, Γ_{2222}, Γ_{1122} equal the elastic constants identically. The Kirchhoff–Love hypothesis $[\Gamma_s^{(1)} = -c_s,\ \Gamma_s^{(2)} = c_s]$ is conserved. Then the quantity $\xi_m + \xi_m^o$ is constant for a fixed load. The shape of the initial deflection agrees with the axisymmetric mode of ideal shell buckling.

We find from (9) the value of ξ_m^o corresponding to the instantaneous ($t_* = 0$) buckling of the axisymmetric equilibrium mode for a given value of the parameter α_m. For a shell with the characteristics $E_{2222} = E_{1111} = E$, $E_{1122} = -\nu E$ in Fig. 1,

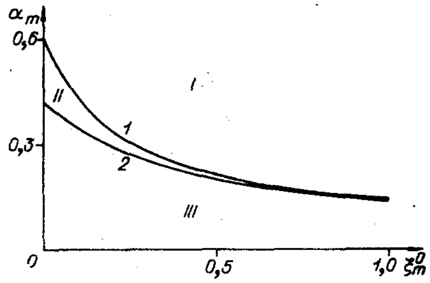

Fig. 1

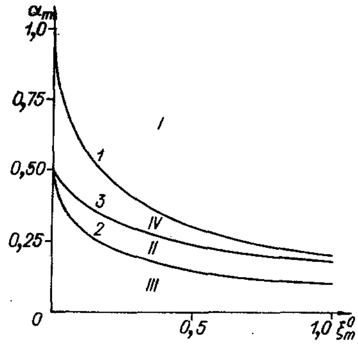

Fig. 2

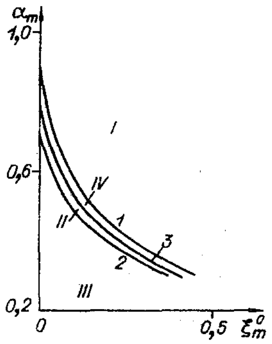

Fig. 3

TABLE 1

1	2	3	4	5
α_m		s, $k=11$	s, $k=9$, 11, 13	s, $k=7$, 9, 11, 13, 15
0,9	0,0480 ($n=11$)	0,0816 ($n=11$)	0,0485 ($n=11$)	0,0485 ($n=11$)
0,8	0,1010 ($n=11$)	0,1869 ($n=11$)	0,1015 ($n=11$)	0,1014 ($n=11$)
0,7	0,1635 ($n=11$)	0,3659 ($n=10$)	0,1642 ($n=11$)	0,1640 ($n=11$)
0,6	0,2406 ($n=11$)	—	0,2418 ($n=11$)	0,2415 ($n=11$)
0,5	0,3419 ($n=10$)	—	0,3420 ($n=10$)	0,3411 ($n=10$)
0,4	0,4785 ($n=10$)	—	0,4807 ($n=10$)	0,4796 ($n=10$)
0,3	0,7171 ($n=9$)	—	0,7221 ($n=9$)	0,7193 ($n=9$)
0,2	1,3415 ($n=7$)	—	1,3586 ($n=7$)	1,3474 ($n=7$)

$$E_{1212}/E = 0.346154,\ \Gamma_{1212}/E = 0.207692,\ \nu = 0.13,\ R/h = 100,$$

curve 1 corresponds to the dependence $\alpha_m \sim \xi_m^0$.

To estimate the shell stability for large values of the time t, we use the property of invariance of the kernels $K_{1212}(t-\tau)$ and $\Gamma_{1212}(t-\tau)$ relative to the origin and we transfer it to $-\infty$. As is known [5], the solution of (7) for a fixed parameter α_m is bounded (constant) for values of ξ_m^0 less than those calculated as roots of an equation similar to (9) and obtained by replacing E_{1212} therein by the creep shear modulus $E_{1212}^{cr} = E_{1212} - \Gamma_{1212}$. For values of α_m and ξ_m^0 which are a root of the equation mentioned, creep buckling ($t_* = \infty$) of the axisymmetric equilibrium mode occurs. Curve 2 in Fig. 1 corresponds to the relationship between these ξ_m^0 and α_m. The mode of such shell buckling can be different from the instantaneous buckling mode because of the different n.

An analysis of the results shows that the shell buckles at the time of load application for α_m, ξ_m^0 belonging to the domain I (see Fig. 1). If a point with the coordinates α_m, ξ_m^0 is taken from domain III, then the shell is stable for any time t. Here Lyapunov stability [6] is conserved. Finally, if a point of the domain II is considered, then the additional perturbation of shell deflection increases without limit for appropriate values of α_m, ξ_m^0 and an unbounded increase in the time. It can be shown that the structure is Lyapunov unstable for the selected parameters α_m, ξ_m^0, but the shell is stable in any previously assigned interval of the time t [1].

Let us note the qualitative agreement between the results and analogous indices for rods. The domain of external load variation for them is also divided into three parts (if the linear problem is solved for any small initial curvatures).

Example 2. The shell is isotropic and the interlayer shear is not taken into account. For simplicity the material is considered incompressible. The mode of the initial deflection agrees with the axisymmetric buckling mode for an ideal shell.

By analogy with the preceding case, we determine the values of the parameters α_m, ξ_m^0 corresponding to the instantaneous and creep bucklings of a shell with the characteristics $R/h = 147$, $\Gamma/E = 0.5$ (Fig. 2, curves 1 and 2).

For values of α_m, ξ_m^0 corresponding to points lying below curve 1, the shell can buckle by a "snap" after the lapse of a certain finite time t_* called the critical time [1, 2]. To obtain t_* it is sufficient to equate the determinant comprised of coefficients of the quantities c_k in (7) to zero by assuming the operators Γ_{ijkl}, K_{ijkl} to be identically equal to the elastic constants, but the quantities ξ_m are considered functions of the time t. Under these conditions an equation is obtained which agrees with (9) or (10).

An analogous equation to seek the critical time for unbounded creep of the material has been obtained in [7] where the criterion of bifurcation of the equilibrium state is used.

The lower boundary of the buckling domain for a finite time t_* corresponds to those values of the parameters α_m, ξ_m^0 for which t_* tends to infinity. The function $\xi_m + \xi_m^0$ has a constant (finite value) $(\xi_m + \xi_m^0)_\infty$ for the α_m, ξ_m^0 considered. Substituting this expression into (10), we obtain the relationship between α_m and ξ_m^0, shown by curve 3 in Fig. 2. Thus, the domain of variation of the parameters α_m, ξ_m^0 is divided into four parts. In addition to those zones which had been determined in the previous example, a buckling zone for the finite time t_* (zone IV) is added.

Example 3. The shell is transversely isotropic. The time-change in just the interlayer shears ($\Gamma_{3131} = \Gamma_{3232} = \Gamma'$) is taken into account.

For such a shell the expressions a_{sn}, α_{sn}, A_{kn}, B_1, B_2 have the form

$$a_{sn} = \frac{n^4h^2\left(1+\theta_{sn}^2\right)^2}{12\left(1-\nu^2\right)R^2\theta_{sn}^4}\left[\left(1+\theta_{sn}^2\right)^2\frac{1-\nu}{2}+\left(1+\theta_{sn}^2\right)\frac{3-\nu}{2}\Lambda+\Lambda^2\right]^{-1}\times$$

$$\times\left[\left(1+\theta_{sn}^2\right)^2\frac{1-\nu}{2}+\Lambda\right]\Lambda+\left(1+\theta_{sn}^2\right)^{-2},\quad \alpha_{sn} = a_{sn}^{-1}\frac{s^2\pi^2}{l^2}\frac{R^2}{Eh}q,$$

$$A_{kn} = \left(1+\theta_{kn}^2\right)^2,\quad \Lambda = \frac{12\left(1-\nu^2\right)R^2}{n^2h^2}\theta_{sn}^2\frac{1}{E}\Gamma',$$

$$B_1 = \left[\theta_{kn}^2+\left(\frac{m}{k}-1\right)^2\right]^2,\ B_2 = \left[\theta_{kn}^2+\left(\frac{m}{k}-1\right)^2\right]^2$$

(ν is the Poisson ratio).

The parameter ξ_m is determined from (4)

$$\left[\frac{m^4\pi^4}{12\left(1-\nu^2\right)}\frac{R^2h^2}{l^4}+1+\frac{m^2\pi^2h^2}{12\left(1-\nu^2\right)l^2}EK'\right]\xi_m -$$

$$-\left[\frac{m^2\pi^2R^2}{l^2}+\frac{m^4\pi^4}{12\left(1-\nu^2\right)}\frac{R^2h^2}{l^4}EK'\right]\frac{q}{Eh}\left(\xi_m+\xi_m^0\right)=0.$$

Curves analogous to curves 1-3 in Fig. 2 (Example 2) are shown in Fig. 3 ($R/h = 40$; $E/E_{3131} = E/E_{3232} = 5$; $E/E_{3131} - \Gamma' = E/E_{3232} - \Gamma' = 50$) for the shell under consideration. In contrast to the two previous cases, in which the bending mode of the shell middle surface remains unchanged in the precritical state, curves 1-3 are constructed for different axisymmetric equilibrium modes in the shell under consideration. The axisymmetric mode of instantaneous ($t = 0$) buckling of an ideal shell corresponds to curve 1 while the axisymmetric mode of creep ($t = \infty$) buckling corresponds to curves 2 and 3. For a fixed amplitude of the initial deflection, the vertical coordinates of points of curves 1-3 are the lower bounds for all values of the parameter α_m, which correspond to instantaneous or creep buckling of a shell possessing a sinusoidal initial deflection.

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